# Overture: Paraconsistent Logics*

João Marcos†
Group for Theoretical and Applied Logic (GTAL)
IFCH – State University of Campinas (Unicamp), Brazil
<vegetal@cle.unicamp.br>
http://www.geocities.com/jm_logica/

August 2002

## 1st: Statement

toccata

Paraconsistency, as the study of inconsistent yet non-trivial logical systems, is a quintessential product of contemporary logic. Being somehow implicit in the doctrine of Western thinkers such as Heraclitus, Hegel and Meinong, as well as some others who have endeavored, in opposition to Aristotle, Plato and Frege —if one is allowed to put so many centuries in the flap of a butterfly's wings—, to develop decent theories in which contradictories could subsist or even be the object of rational study, a concern about the study of logical systems that could sit in the background of possibly contradictory theories was more than ever on the rise at the very same time in which the logicist and formalist schools in the philosophy of mathematics were under constitution, about a hundred years ago. Indeed, the Polish logician Jan Łukasiewicz[1] and the Russian Nikolai A. Vasiliev[2] both composed in 1910 some firm criticisms of the most classical of logical principles, the first of these authors concentrating on the various formulations under which he would propose to understand Aristotle's views on contradiction, the second author again criticizing those same views and proposing a revolutionary approach to logic inspired on the 'imaginary' geometry of his university fellow Nikolai Lobachevsky. In the manner of Lobachevsky, who

*This introductory movement is meant as an opening aria for the *Workshop on Paraconsistent Logic* (WoPaLo), http://logica.rug.ac.be/WoPaLo/, held in Trento, Italy, from 5 to 9 August 2002, as part of the XIV *European Summer School in Logic, Language and Information* (ESSLLI 2002), http://www.esslli2002.it/, organized under the auspices of the European Association for Logic, Language and Information (FoLLI), http://www.folli.uva.nl/.

†With thanks to Dirk Batens and Walter Carnielli, both of which having been scientific maestri for both this workshop and myself. My work on this was initially supported by the Research Fund of Ghent University, in Belgium, project BOF 2001/GOA/008, and later by a CNPq fellowship, in Brazil, proc. 140732/2002-0. Thanks are also due to the Organizing Committee of the ESSLLI 2002 and their impresarios, for all help given.

[1]In "On the principle of contradiction in Aristotle", appearing as a book in Cracow, in Polish, and in abridged form as a paper, in German, in the *Bulletin International de l'Académie des Sciences de Cracovie*.

[2]In "On partial judgements, the triangle of opposition and the law of excluded fourth", appearing in Russian, in the *Učënie Zapiski Kanzan'skogo Universitéta*, in Kazan.

had tried his hand on non-Euclidian geometry more than eighty years before, investigating the independence of the postulate of parallels, Vasiliev forged his own effect of dissonance by investing on non-Aristotelian logic, investigating the independence of the operation of negation —the naive notions of parallelism and of negation would never be the same, after that.[3] The efforts of Łukasiewicz and Vasiliev would mark thus the start of some serious work on non-classical, 'imaginary', logic —notably many-valued logics and dialectical logics—, while paraconsistent logics rested just a bit further ahead.

`vivace`

The actual birth of paraconsistent logics can arguably be placed a few decades after those aforementioned exploring études, just after World War II, when Łukasiewicz's disciple Stanisław Jaśkowski[4] and the Brazilian logician Newton C. A. da Costa[5] both argued independently for the development of logical systems in which the classical principle whence anything follows from a contradiction would be put under control. So, as those pioneers phrased with precision, the point now would rather be to avoid *triviality* —anything following from anything else— by taming the explosive character of contradictions in classical or other consistent logics. This constituted, in one way or another, a direct and effective response to Wittgensteinian counsels about toleration of contradictions, just a few years before, to the effect that inconsistency should be no more "superstitiously feared or awed by mathematicians". The approach of Jaśkowski was mainly targeted at the formalization of situations of discussion, when usually not all opinions are in agreement. The wideness of this original formulation and its underlying semantical intuitions —exploring the outcome of the conjoining of different sources— can quite consistently be said to descend from those of Vasiliev and somehow lie behind practically every other approach to paraconsistency later on. Da Costa's motivations comprised primarily the development of systems that would be strong enough as to allow for the reproduction of the most part of 'normal' mathematics, while at the same time would avoid the paradoxes that had marked the logical fields forever, especially since the beginning of the XX century. The (still underdeveloped) impact of paraconsistent logics on the foundations of mathematics and its modern import in computer science owe much to this last approach and its forthcoming fruits.

`accelerando`

There is no space for warbling here the extraordinary thematic development of paraconsistency from then on. Further independent but a bit less influential tunes —as yet— came along, in the two decades surrounding the exposition of the pioneers: such opuses included for instance the investigations of Florencio Asenjo and David Nelson. The 70s then gave ear to the translations of the first papers into English, followed by a multiplication of the interest on and an extension of the reach of their proposals, and by the emergence of several new approaches and techniques. Also, a whole bunch of other authors kept on reinventing paraconsistency over and over again. The 80s observed mainly the organization of various schools around those proposals and the vulgarization of the old and the new developments in the area, plus of course some more reinvention of the same. In the last decade several significative congresses,

[3] More on the resemblance of the two Nikolais' programs can be found in V. A. Bazhanov's "The imaginary geometry of N. I. Lobachevsky and the imaginary logic of N. A. Vasiliev", *Modern Logic* **4**:148–156, 1994.

[4] In "Propositional calculus for contradictory deductive systems", published in Polish, in 1948, in *Studia Societatis Scientiarum Torunensis*.

[5] In "Inconsistent Formal Systems", his cathedra thesis, in Portuguese, UFPR, 1963.

meetings and workshops on or closely related to paraconsistency were orchestrated: the I *World Congress on Paraconsistency*[6] was conducted in Ghent, Belgium, in 1997; this was soon followed by the *Stanisław Jaśkowski Memorial Symposium*,[7] in Toruń, Poland, in 1998; then came the II *World Congress on Paraconsistency*,[8] in Juquehy, Brazil, in 2000; two *International Workshops on Living With Inconsistency*[9] animated people from USA and Canada in 1997 and 2001; four *Flemish-Polish Workshops on the Ontological Foundations of Paraconsistency*[10] were held in between 1999 and 2001; the I *International Workshop on Computational Modelling of Scientific Reasoning and Applications*,[11] in Las Vegas, 2001, was also dominated by the theme; a workshop on *Inconsistency in Data and Knowledge*[12] happened in Seattle in that same year; the present year of 2002 is testimony to this *Workshop on Paraconsistent Logic*[13] as well as to the *Workshop on Paraconsistent Computational Logic*,[14] in Copenhagen; there are already expectations about a III *World Congress on Paraconsistency*[15] under rehearsal in Toulouse for 28–31 July 2003. May the excitement in appraising the fruits of paraconsistency not be overestimated: The meetings on paraconsistency have traditionally congregated partisans and detractors; criticisms of paraconsistent logics have usually shown pros and cons, always under a cordial and bountiful scientific atmosphere.

glissando

Hundreds of papers on paraconsistent logic pervaded the literature in these last years, and this eventually led the Mathematical Reviews, in 1991, to start a new section (03B53) with the name "paraconsistent logic" —as it came to be christened in the 70s, by F. Miró Quesada, until recently the president of the International Federation of Philosophical Societies. This imbroglio would evolve ten years later into the presumably more general description "logics admitting inconsistency (paraconsistent logics, discussive logics, etc.)" for the same section of the Mathematics Subject Classification. Such rapidly increasing Klangfarbenmelodie would rapidly take over and give way to some books and theses as well —a suite of those can be found in the coda of this overture.

[6] Check `http://logica.rug.ac.be/centrum/events/WCP97/index.html`.
[7] Check `http://www.uni.torun.pl/~logic/JS'98/`.
[8] Check `http://logica.cle.unicamp.br/wcp/wcp2000.htm`.
[9] Check `http://recluse.ics.uci.edu/pub/icse-97/workshop/inconsistency.html` and `http://www.cs.toronto.edu/~sme/IWLWI-01/`.
[10] Check `http://logica.rug.ac.be/centrum/events/Vlapol2/2deworkshop.htm` and `http://logica.rug.ac.be/centrum/events/Vlapol4/4deworkshop.html`.
[11] Check `http://www.lip.uns.edu.ar/cmsra/`.
[12] Check `http://www.cse.buffalo.edu/~chomicki/inconsistency01.html`.
[13] Check the opening note.
[14] Check `http://floc02.diku.dk/PCL/`.
[15] Check `http://www.irit.fr/wcp3/`.

## 2nd: Call and response

idée fixe

The history of pedagogy has it that the Seven Liberal Arts have been extremely influential in the classical curricula at least since the Middle Ages. Arranged in a group of *artes triviales*, which consisted of Grammar, Rhetoric and Logic, and a group of *artes quadriviales*, which consisted of Arithmetic, Geometry, Music and Cosmology, the seven *artes liberales* were called as such as they served the purpose of training the free man. The new *trivium* successfully consecrated in the last years by the summer schools on logic, language and information substituted Computer Science for Rhetoric, and revitalized thus the old formula, inflating it with new scientific interest and introducing at the same time some element of *artes illiberales* —those that are pursued for economic purposes.

encore

In the very same spirit, paraconsistency has gained quite some freshness in affirming itself as a perfect crossover. Being primarily targeted in its initial phase to deal with issues related to the formalization of both the ordinary and the scientific discourses, as well as matters related to the foundations and the philosophy of mathematics, paraconsistent logic grew its pseudopods into more applicable directions, as one can easily check by a quick perusal of the literature and of the events that involved this sort of logic in the last so many seasons.

da capo

Besides bringing, since its early years, new ideas into the semantics, the syntax and the pragmatics of logic, paraconsistency would soon come to affect critical thinking, argumentation theory, and some deeply rooted practical-philosophical notions and attitudes, forcing a reexamination even, as it has been argued, of the received classical account of rationality. In a world overloaded with information at the most diverse fields of human activity —and, even more, of machine activity— the need for the development of reasoning mechanisms tailored to meet the minimally rational requirements of inconsistent yet non-trivial theories that pop up in those fields turned out to be of capital importance, and not rarely a matter of survival. The interest in the investigation of methods which would help us avoiding, revising or tolerating the occurrence of contradictions *de dicto* or *de facto* in the theories we are bound to work with has been accompanied by the increasing popularity and success of the paraconsistent enterprise. All the necessary and sufficient conditions being given, the paradigm of paraconsistency has purportedly occupied its territory and touched numerous other branches of logic, plus some philosophical areas from ontology to metaphysics, from epistemology to history, methodology and philosophy of science, not to mention some assays ventured into theoretical modern physics, psychoanalysis and linguistics. Its rationale, its range of application, its practical and philosophical status and import, however, all seem to rest still largely open to dispute and regulation.

moderato

Paraconsistent logics were proposed both as alternative and as supplementary to consistent logics such as intuitionistic logic and its extensions, while at the same time they struggled to get the right of being studied on a par with the latter, and with other non-classical logics. As provisional or definitive substitutes to other logics in situations in which the domain is not known or in principle not knowable to be consistent, paraconsistent logics have done a great deal of effort, in many cases, to show that the forms of inference practiced by those other logics could still be recaptured inside these latter-day non-trivially inconsistent or possibly inconsistent domains. In practice, paraconsistency started to find its way into corrective, ampliative,

plausible, hypothetical, probabilistic, analogical, inductive, abductive, causal, explanatory, argumentative and defeasible forms of reasoning, besides procedures in which hypotheses get formed, learning gets effected and discoveries get made. Behind all that, the instigating paradoxes always lay in waiting...

`legato` Later on, issues connected to computer science initiated a reshaping of the paraconsistency scenario that would change its face once and for all —or *one* of its faces, as it has often been compared to a Janus-like creation. While a human agent would hardly willfully admit indifferent conclusions to be drawn from some given premises which turned out to be contradictory after detailed analysis, one should certainly not be so optimistic as to expect the same principled behavior and commonsense from automated agents. (And remember that, in a consistent environment, one single contradiction is already painfully explosive.) Now, automated reasoning research in software engineering, artificial intelligence or database management has early learned to see inconsistency, all sorts of it, as just one of those facts of life. Wailing and gnash of teeth won't help much in the furnace of fire of yet another unpredicted loop and deep-blue screen, or that nice seemingly alleatoric behavior of unintentionally produced incredibly complex cellular automata! Seriously speaking, it is surely not a surprise that the computational logic community has been steadily increasing its awareness of the difficulties and the urgency of the matter of inconsistency handling. It has in fact been argued that the fact that most computing systems in practice usually *are* able to provide meaningful information, despite ubiquitous inconsistencies, reflects a mismatch between theory and practice, raising thus a profound challenge to the legitimacy of logical foundations of computing in general (follow for instance the link on note 14). The pressing task thus would be to build a bridge between theoreticians and practitioners of paraconsistency, in the field of computer science.

`andante` Model checking, theorem proving, logic programming, and model-based reasoning, among other tasks, cannot simply be paralyzed in the fear of a contradiction. Prevention, detection, tolerance and resolution should each be tried at its turn. Contradiction removal is not always feasible or cheap. Providentially designing your tools in an appropriate fashion gives often a higher benefit at a lower cost. Inconsistency is no more a bug than consistency a virtue. Meanwhile, the need to find sensible ways of dealing with contradictory specifications prods on and on again, as constraint solving and integrity maintaining keeps worrying the database administrators as much as ever, abstraction, representation, and integration of different sorts and huge amounts of information calls for smarter knowledge engineers and their wondrous gadgets. On issues as lively, diverse and advanced as data mining, evolutionary computation or semantic web, the idea of conjoining viewpoints and the notion of independent (self-organizing) agents working on cooperative and distributed environments are remarkable vindications of Jaśkowski's early discussive ideas and of the paraconsistent unfolding that followed. In order to model the dynamics of knowledge in flux, while at the same time allowing for the intended chain of reasoning to occur as consistently as possible even in the presence of inconsistencies, operations of (belief) revision and update are often effective in the resolution of conflicts that show up. The fallout of contradictions might be circumscribed (with an eye on runtime behavior), or a preference relation between models might be introduced, and again one is able to reconstruct usual consistent logics such as classical logic

'unless and until' there is evidence to the contrary —the sum and substance of defeasible reasoning.

ritornando

Summing the gains and losses, there are some general lessons to be learned from the paraconsistent rhythm, harmony and melody this far. It should be absolutely clear, for instance, that the nature of the enterprise is connected to the failure of certain *inferences*, more than the breakdown of particular theses or theorems. The possibility, significance and contribution of paraconsistent logics is founded in spotlighting the misidentification of the events of contradictoriness and of triviality of a theory. Indeed, if explosiveness of contradictions is controlled, there is no reason not to allow for inconsistent yet non-trivial logics to appear! At least three interesting definitions of paraconsistent logic have found their way into the literature. Jaśkowski's definition called for the investigation of logics that could serve as bases to non-explosive theories, that is, theories which would not be trivial —inferring everything else— even when considered in the presence of contradictions. Da Costa's definition called for the investigation of logics that could underlie contradictory theories while at the same time prevented some of them to be trivial. A third general definition considers that consistent logics are those which are both explosive and non-trivial, and calls for the investigation of paraconsistent logics exactly as those which are inconsistent yet still non-trivial. The equivalence of the three definitions can be shown under appropriate qualifications about the properties of the underlying inference relations of the considered logics.[16]

perpetuum mobile

Further impetus has been gained by the paraconsistent saltimbanchi when the development by continuous imitation of modern non-classical logics lent further and further support to their presentations. Starting from different programmes, several distinct classes of non-classical logics arrived to paraconsistency as a particular, but sometimes fundamental, case of application. Thus, substructural logics, in allowing for the absence of structural rules present in classical logic, have been particularly rich of examples of rediscovery of paraconsistent concerns and results. Non-trivial dialectical logics are paraconsistent. Minimal intuitionistic logic is only partially explosive, being paraconsistent in a weak sense. Relevance logics, in the search for a good conditional and for inference relations that would reflect the existence of some relation between the content of the premises and that of the conclusions, have consequently prohibited explosion to occur. Some operators of some linear and quantum logics can easily be shown to behave paraconsistently —with respect to certain convenient inference relations. (Some modal extensions of classical logics also can define such kind of operators.) In the case of paraconsistent non-monotonic logics, basically two staffs can be followed —to get sometimes roughly the same results: one can revise a given set of premises in order to render it consistent, and then draw conclusions; or else one can keep the given set of premises, and restrict instead its set of conclusions.[17] And so on, and so forth.

[16] Have a look at our paper "A Taxonomy of **C**-systems", in [2002], for precise definitions of all the above terms, and for careful distinctions drawn in between contradictoriness and inconsistency, the principles of consistency, explosion, non-trivialization and non-contradiction.

[17] A particularly interesting example of this last solution is the one practiced by the first known *adaptive logics*. Have a look at the paper "A survey of inconsistency-adaptive logics", in [2000b].

## 3rd: Double exposition

The current commitment to a certain doctrine of the affections has led us this far to a blitzsurvey of paraconsistent logic, its origin, development and worthiness. It is time now to close by referring to the melodies to come, quickly explaining more or less what you will find in the libretto hereby opened. Of course, choices have to be made in explaining the material, and the brief descriptions presented here certainly cannot substitute the richer experience of an audition of the full respective papers. pizzicato

The present workshop was organized inside the Logic & Computation section of the summer school, aiming to provide a forum for advanced Ph.D. students and other researchers to present and discuss their work in the area. The leitmotif was pretty clear: paraconsistent logic. The submission guidelines that were publicized about a year ago solicited the dispatching of extended abstracts which could help providing to the participating students of the summer school a good survey of the most interesting and promising recent research in the area. Ideal lectures were defined as those which contained (i) a good motivation (either with respect to a domain of application or with respect to a theoretical problem —the former seeming more desirable given the audience), (ii) a clear statement of the problem that is solved by the paper, (iii) a clear presentation that is both understandable for a wider audience and meets the highest technical standards. In selecting among the submissions the papers which should be presented we had the splendid help of our referees.[18] The workshop should take 1h45min in each of the five afternoons, and each session is to be closed by an invited talk. It should be observed, as usual, that the opinions and ideas expressed in the forthcoming divertimento, for the good and for the bad, are of exclusive responsability of their authors. Any misinterpretation of their works in the brief descriptions below is of course my own fault. allegro ma non troppo

In "Models for a paraconsistent set theory", Thierry **Libert** searches for natural models for paraconsistent fragments of classical set theories. Among the various possible solutions which he nicely surveys, some emphasis is put on paraconsistent extensions of *positive set theory*, in order to circumvent traditional difficulties such as Russell's paradox. Paraconsistent sets are actualized using a sort of Vasiliev's trick, considering membership and non-membership as somewhat independent properties. A measure of inconsistency is introduced in the hunt for minimally inconsistent models. arpeggio

Next, in "Diamonds are a philosopher's best friends. The knowability paradox and modal epistemic relevance logic", Heinrich **Wansing** gives a very clear exposition of a *paradox of epistemic logic* originating in an old paper by Fitch, to the effect that the knowledge of every true statement by a cognitive agent follows from the mere *possibility* of knowing each such true statement. A solution to please even anti-realists is offered in the spirit of Nelson's approach to constructivism combined with an implication coming from relevance logic. A convenient system is built to avoid the paradox, and defined both axiomatically and semantically.

[18] Namely, Ofer Arieli, Arnon Avron, Diderik Batens, Valentin Bazhanov, Leopoldo Bertossi, Carlos Caleiro, Walter Carnielli, Marcelo Coniglio, Sandra de Amo, Marcelo Finger, Melvin Fitting, Larisa Maksimova, João Marcos, David W. Miller, Chris Mortensen, Hiroakira Ono, Graham Priest, Greg Restall, Frank T. Sautter, Luis A. Sbardellini, Ralf Schweimeier, Guido Vanackere, and Heinrich Wansing.

Sergei P. **Odintsov**, in "On the structure of paraconsistent extensions of Johansson's logic", reports systematically on his long-term lattice-theoretic work on logics that are paraconsistent yet *partially explosive*. An interesting class of paraconsistent intermediate logics, extensions of minimal intuitionistic logic, is then defined from its classes of algebraic counterparts that are neither Boolean nor Negative. As in the general paraconsistent case, contradictions which appear there are not necessarily equivalent. Kripke semantics for such logics are also concisely described.

In "Using paraconsistent models in logic program verification", Paolo **Mascellani** argues for the paraconsistent behavior (with respect to some presupposed semantical inference relations) presented by a class of negations commonly used in computational logics, as for instance in *program verification* and *termination analysis*. The author also hints enticingly, together with some other sources in the literature, to the usefulness of allowing for inconsistent models to be considered, instead of just the consistent ones, and to the convenience of working with four-valued models instead of three-valued ones.

In "How to build your own paraconsistent logic: An introduction to the Logics of Formal (In)Consistency", Walter A. **Carnielli** shows how the very *notion of consistency*, the cornerstone of paraconsistency, can be internalized into the object language level, characterizing thus a class of gently explosive logics which generalize the approach of da Costa. The remarkable advantages which derive include an automatic way of recovering consistent reasoning inside paraconsistent environments, and the setting of a coherent organizational framework for the classification and study of existing paraconsistent logics —most of which, in fact, are already straightforwardly characterizable as gently explosive. The author surveys in some detail a few important logics belonging to this category, and illustrates their interpretation and use in computer reasoning.

Greg **Wheeler** starts his "Statistical defaults and paraconsistency" by a clear presentation of default logic, followed by a proposal of formalization of the concept of *statistical default*. He argues then for the non-monotonic behavior of statistical inference, and sets a plot which will motivate very naturally the worries about inconsistency toleration. Several examples are considered in order to show how such preoccupations are imperative, and an initial preferred solution is then sketched. The intention, again, is to characterize an interesting logical problem that deserves further attention, at least for those working in empirical branches of science.

"Paraconsistent informational logic", by Paola **Forcheri** and Paolo **Gentilini** stresses an application of paraconsistent logics to formal epistemology. The authors present a formalism to express conjectures as formal objects, where the deductive apparatus of *conjecturing agents* is conflated with some given environment system. Here, in this interaction of agents with environment, inconsistencies might quite reasonably emerge. According to a logical entropy measure they introduce, a theory which contradicts another one can still constitute a very good conjecture with respect to the latter, and this clearly extends the notion of rationality of an agent holding such a theory. The formalism is then applied for a particular gently explosive logic, and a proof-theoretical investigation follows.

In "Some computational aspects of inconsistency-adaptive logics", Diderik **Batens** investigates *goal-directed proofs* for adaptive logics —logics characterized by inference relations that lack a positive test, very common in both quo-

tidian and scientific reasoning processes. As the best one can have in such cases are fallible but sensible estimates of (final) derivability, the author searches for a proof format that functions in itself as a criterion for derivability. One must still decide, of course, in each stage of a proof, whether one will continue the proof or rely on the insights one might already have at that stage, which will never be worse than those one had at previous stages. The new informative proof procedure, indeed, is developed by the author in order to incorporate more decidability criteria directly into the proofs.

Vladimir **Vasyukov**'s study "Paraconsistency in categories: case of relevance logic", delves into the paraconsistent behavior of some *categorial models of relevance systems*. Building on previous investigations, the author bases his proposed categorial semantics in a construction of toposes composed of functors from an adequate relevance preorder category —reformulating the relevance algebra with appropriate functors for negation and implication— into the category of sets.

"Paraconsistent logic from a modal viewpoint", by Jean-Yves **Béziau**, offers a *modal perspective* on paraconsistent negation which is distinct both from Jaśkowski's approach to paraconsistency in terms of **S5** and from an old alternative four-valued modal logic defended by Łukasiewicz. Indeed, a lively new approach based on **S5** and a brand new four-valued logic with modal flavor are presented by the author, and their respective negation operators are defined based on the idea of contrasting necessity and 'non-necessity', in much the same way as a contrast of possibility and impossibility has produced a negation with intuitionistic flavor in previous works of Došen.

The set of slides on "Reasoning with inconsistency in structured news reports", by Anthony **Hunter**, shows a pleasant application of tricks of paraconsistency in automating *reasoning over structured texts*. The basic example is that of news reports, tagged with semantic labels, represented and organized in such a way as to facilitate treatment by logic-based techniques, aided by convenient specialized domain-oriented knowledge. A sort of quasi-classical logic is sketched using models with positive and negative objects, in the line of Vasiliev's approach, and heterogeneous reports are merged, analyzed and used in argumentation in a way similar to that of Jaśkowski. Inconsistencies are in principle considered as informative. Four-valued models and a measuring for inconsistency are motivated, and introduced in order to minimize conflict and reduce the set of models, thus strengthening the results of the argumentation process.

In "Paraconsistent logic programs", João **Alcântara**, Carlos Viegas **Damásio** and Luis Moniz **Pereira** illustrate the use of *bilattices in logic programming* for reasoning with uncertain, incomplete and inconsistent information, in a way that immediately appeals to those who are familiar with the work of Fitting and with probabilistic deductive databases. Also here, Vasiliev's trick of considering positive and negative evidences is used in the background. As motivation, several interesting examples of the large range of applications of the authors' techniques are contemplated. The framework provided is argued to be strong enough as to allow for the embedding of other logic programming systems.

The author of "Classical, intuitionistic and paraconsistent logic in scientific theories", Antonino **Drago**, considers axiomatic and methodological presentations of scientific theories, and then dissertates about the role of inferences of *double negated elimination*, which would not be fully valid in the latter. The

main links with paraconsistency here lie in its duality with intuitionistic logic and in relying on Vasiliev's heuristic approach to non-Aristotelian logics.

In "Combining paraconsistency and relevance", Arnon **Avron** explores relevance concerns inside paraconsistent domains. Many-valued logics with non-equivalent intermediary 'paradoxical' values and written in the language of purely intensional relevance logic are examined, based on criteria of *symmetry* (there should be no way of distinguishing between two given paradoxical values on a purely logical basis) and *isolation of contradictions* (paradoxical values are assigned to a formula if and only if all its constituents are assigned that same paradoxical value). The outcome is contrasted to some gently explosive logics in the literature, and strongly adequate cut-and-contraction-free formulations of the first logics in terms of hypersequents are offered.

With some luck, the diversity of the papers delivered in this Gesamtkunstwerk can be thought of as quite representative of the variegated research in the field of paraconsistent logics. I hope this sounds like music for you.[19] `resolution`

## Collegium Musicum[20]


[2002] *Paraconsistency: The logical way to the inconsistent*, Walter A. Carnielli, Marcelo E. Coniglio, and Itala M. L. D'Ottaviano (editors), Marcel Dekker, 2002.

[2000b] *Frontiers of Paraconsistent Logic*, Diderik Batens, Chris Mortensen, Graham Priest, and Jean-Paul van Bendegem (editors), Taylor and Francis, 2000.

[2000a] *Semânticas de Traduções Possíveis*, João Marcos, Thesis, Unicamp, 2000. `http://www.cle.unicamp.br/students/J.Marcos/index.htm`

[1998b] *Elementos de Teoria Paraconsistente de Conjuntos*, Newton C. A. da Costa, Jean-Yves Béziau, and Otávio Bueno, Coleção CLE, 1998.

[1998a] *Wahre Widersprüche: Einführung in die parakonsistente Logik*, Manuel Bremer, Academia Verlag, 1998.

[1997] *Ensaio sobre os Fundamentos da Lógica*, Newton C. A. da Costa, Hucitec, 1994. Translated as *Logiques Classiques et Non Classiques*, Masson, Paris 1997.

[1996] *Inconsistencias ¿Por Qué No? Un estudio filosófico sobre la lógica paraconsistente*, Andrés Bobenrieth Miserda, Santafé de Bogotá, Tercer Mundo Editores, Colcultura, 1996.

[1995] *Beyond the Limits of Thought*, Graham Priest, Cambridge University Pr., 1995.

[1994] *Inconsistent Mathematics*, Chris Mortensen, Kluwer, 1994.

[1993] *Sistemas Formais Inconsistentes*, Newton C. A. da Costa, Thesis, 1963. Editora UFPR, Curitiba, 1993.

[1989] *Paraconsistent Logic: Essays on the inconsistent*, Graham Priest, Richard Routley, and Jean Norman (editors), Philosophia, 1989.

[1987] *In Contradiction: A study of the transconsistent*, Graham Priest, Martinus-Nijhoff, 1987.

[1983] *Logica Paraconsistente: Una introduzione*, Nicola Grana, Naples, Loffredo Editore, 1983.

[1980] *The Logic of Inconsistency: A study in nonstandard possible-world semantics and ontology*, Nicholas Rescher, and Robert Brandom, Basil Blackwell, 1980.


[19] Some further helping links for those who have just arrived in the concert are: `http://plato.stanford.edu/entries/logic-paraconsistent/` and `http://plato.stanford.edu/entries/mathematics-inconsistent/`, for some entries on *Paraconsistency* and on *Inconsistent Mathematics;* `http://groups.yahoo.com/group/paraconsistency` for the *Paraconsistency Discussion List*.

[20] Links to this set of works can be found at `http://logica.rug.ac.be/WoPaLo/books.htm`.