Proof complexity and analytic cuts in finite-valued logics

Christian G. Fermüller[1] and João Marcos[2,1]

[1] Theory and Logic Group, Institut für Computersprachen, TU-Wien
[2] LoLITA and Department of Informatics and Applied Mathematics, UFRN, Brazil

It is well known that proofs involving the cut rule (or, equivalently, modus ponens) can be dramatically shorter than the shortest cut-free (sequent or analytic tableau) proof of the same assertion. On the other hand it is obvious that an unrestricted use of cuts may lead to infinitary branching in proof search. Taking those facts into account, restricted forms of cut have been investigated that imply gains in minimal proof size without rendering proof search unwieldy. In particular, cut-based tableaux (cf. [5, 6]) are based on a goal-directed form of employing *analytic cuts*, i.e., cuts involving formulas that are (closely related to) subformulas of the target formula. D'Agostino (cf. [4]) proved that (propositional) cut-based tableaux polynomially simulate the truth-table procedure while the shortest standard analytic tableaux simulations are exponentially larger in the worst cases.

The aim of this contribution is to investigate to which extent the above described findings for classical logic generalize to the realm of finite-valued logics. The situation is less immediate than might be supposed, in particular because quite different, competing approaches to generic proof search for finite-valued logics are on the market. We will focus on two methods: sets-as-signs tableaux (cf. [7, 1]), where formulas are labeled by sets of truth-values; and classic-like tableaux (cf. [2, 3]), where one recovers bivalence with the help of expressive formulas of the (possibly conservatively extended) target logic in order to account for the multiple truth-values in a 2-signed setting. In the latter case one can directly take over classical cut, but has to deal with a multiplication of the number of rules, due to the syntactic encoding of the truth-tabular semantics. Moreover one might have to design a strategy for their application, to guarantee termination of the associated decision procedure. In the former case all rules for decomposing logically complex formulas can be directly read off from the truth-tables, but a number of different realizations of cut are definable, all corresponding to the fact that a formula can only take a single truth-value in a given interpretation.

Our contribution reports on work on progress. We sketch the required technical background and present a few preliminary results.

References

1. Matthias Baaz, Christian G. Fermüller, and Gernot Salzer. Automated deduction for many-valued logics. In J. A. Robinson and A. Voronkov, editors, *Handbook of Automated Reasoning*, pages 1355–1402. Elsevier and MIT Press, 2001.

2. Carlos Caleiro, Walter Carnielli, Marcelo E. Coniglio, and João Marcos. Two's company: "The humbug of many logical values". In J.-Y. Béziau, editor, *Logica Universalis*, pages 169–189. Birkhäuser Verlag, Basel, Switzerland, 2005.
3. Carlos Caleiro and João Marcos. Classic-like analytic tableaux for finite-valued logics. In H. Ono, M. Kanazawa, and R. de Queiroz, editors, *Proceedings of the* XVI *Workshop on Logic, Language, Information and Computation* (WoLLIC 2009), held in Tokyo, JP, June 2009, volume 5514 of *LNAI*, pages 268–280. Springer, 2009.
4. Marcello D'Agostino. Are tableaux an improvement on truth-tables? cut-free proofs and bivalence. *Journal of Logic, Language, and Information*, 1:235–252, 1992.
5. Marcello D'Agostino. Tableau methods for classical propositional logic. In M. D'Agostino, D. M. Gabbay, R. Hähnle, and J. Posegga, editors, *Handbook of Tableau Methods*, pages 45–123. Kluwer, 1999.
6. Marcelo Finger and Dov M. Gabbay. Equal rights for the cut: Computable non-analytic cuts in cut-based proofs. *Logic Journal of the IGPL*, 15(5–6):553–575, 2007.
7. Reiner Hähnle. Tableaux for many-valued logics. In M. D'Agostino, D. Gabbay, R. Hähnle, and J. Posegga, editors, *Handbook of Tableau Methods*, pages 529–580. Springer, 1999.