► C. BLASIO, J. MARCOS and H. WANSING, *Monotonic functions are logically four-valued.*
IFCH / UNICAMP, Campinas–SP, Brazil.
E-mail: carolblasio@gmail.com.
DIMAp / UFRN, Natal–RN, Brazil.
E-mail: jmarcos@dimap.ufrn.br.
Dept. of Philosophy II / RUB, Bochum, Germany.
E-mail: Heinrich.Wansing@rub.de.

A monotonic function on a set $\mathcal{S}$ is a $\subseteq$-preserving mapping on $2^{\mathcal{S}}$, that is, a function C such that $\mathsf{C}(A) \subseteq \mathsf{C}(A \cup B)$, for every $A, B \subseteq \mathcal{S}$. Tarski's fixpoint theorem guarantees the existence of the least and of the greatest fixpoints for monotonic functions. The latter have a variety of applications, in particular in providing a foundation for inductive and co-inductive definitions, and the proof methods associated therewith. A Tarskian closure operator on $\mathcal{S}$ is a monotonic function on $\mathcal{S}$ that is also inflationary (i.e. $A \subseteq \mathsf{C}(A)$) and idempotent (i.e. $\mathsf{C}(\mathsf{C}(A)) = \mathsf{C}(A)$); it is a generalization of the notion of topological closure, axiomatized by Kuratowski. A closure operator on $\mathcal{S}$ is called structural when it commutes with endomorphisms on $\mathcal{S}$. (Structural) Tarskian closure operators are known [3] to be characterizable by a family of so-called logical matrices, viz. structures containing sets of 'algebraic' truth-values, some of which are distinguished. Their inflationary and idempotent character also guarantees that they may be characterized by (at most) two 'logical' values (cf. Chap. 4 of [4]). In the present contribution we will show how a generalized notion of closure and a two-dimensional notion of logical matrix (resp. B-closure and B-matrix) may be used to characterize any given monotonic function on a set $\mathcal{S}$, recovering a theme earlier explored at [2] in the context of symmetrical consequence relations involving two potentially distinct languages. We will also show that any B-matrix may be alternatively characterized by (at most) four logical values [1]. A brief discussion of inferential many-valuedness and its connections with bilattice-based reasoning, from a metalogical perspective, will ensue.

[1] Carolina Blasio and João Marcos and Heinrich Wansing, *An inferentially many-valued two-dimensional notion of entailment*, to appear in the ***Bulletin of the Section of Logic***, 2017.

[2] Lloyd Humberstone, *Heterogeneous logic*, ***Erkenntnis***, vol. 29 (1988), pp. 395–435.

[3] Ryszard Wójcicki, *Some remarks on the consequence operation in sentential logics*, ***Fundamenta Mathematicae***, vol. 68 (1970), pp. 269–279.

[4] Grzegorz Malinowski, ***Many-Valued Logics***, Oxford, 1993.