Adaptive fuzzy sliding mode control and its application to underwater robotic vehicles

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Abstract

This work presents a discussion about the convergence properties of a variable structure controller for uncertain single-input-single-output nonlinear systems. The adopted approach is based on the sliding mode control strategy and enhanced by an adaptive fuzzy algorithm to cope with modeling inaccuracies and external disturbances that can arise. The convergence of the tracking error vector is analytically proven using Lyapunov's direct method and Barbalat's lemma. An application of this adaptive fuzzy sliding mode controller to an underwater robotic vehicle is introduced to illustrate the controller design method. Numerical results are also presented in order to demonstrate the control system performance.

Palavras-chave

Adaptive algorithms, Fuzzy logic, Nonlinear control, Underwater robotic vehicles, Sliding modes.

Introduction

Sliding mode control, due to its robustness against modeling imprecisions and external disturbances, has been successfully employed to nonlinear control problems. But a known drawback of conventional sliding mode controllers is the chattering effect. To overcome the undesired effects of the control chattering, Slotine [16] proposed the adoption of a thin boundary layer neighboring the switching surface, by replacing the sign function by a saturation function. This substitution can minimize or, when desired, even completely eliminate chattering, but turns *perfect tracking* into a *tracking with guaranteed precision* problem, which actually means that a steady-state error will always remain. In order to enhance the tracking performance inside the boundary layer, some adaptive strategy should be used for uncertainty/disturbance compensation.

Due to the possibility to express human experience in an algorithmic manner, fuzzy logic has been largely employed in the last decades to both control and identification of dynamical systems. In spite of the simplicity of this heuristic approach, in some situations a more rigorous mathematical treatment of the problem is required. Recently, much effort [1, 5, 6, 9, 14, 17, 19] has been made to combine fuzzy logic with sliding mode methodology.

In this work, an adaptive fuzzy sliding mode controller (AFSMC) is proposed to deal with imprecise single-input-single-output nonlinear systems. The adopted controller is primarily based on the sliding mode control methodology, but a stable adaptive fuzzy inference system is embedded in the boundary layer to cope with structured (or parametric) uncertainties, unstructured uncertainties (or unmodeled dynamics) and external disturbances. Using Lyapunov's second method (also called Lyapunov's direct method) and Barbalat's lemma, the convergence properties of the tracking error vector is analytically proven. Based on the proposed control scheme, a depth regulator is introduced for remotely operated underwater vehicles to illustrate the controller design method. Numerical results shows that, when compared with a conventional sliding mode controller, the AFSMC shows an improved performance.

Adaptive fuzzy sliding mode controller

Consider a class of nth-order nonlinear systems:

$$x^{(n)} = f(\mathbf{x}) + b(\mathbf{x})u + d \tag{1}$$

where u is the control input, the scalar variable x is the output of interest, $x^{(n)}$ is the n-th time derivative of x, $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]$ is the system state vector, d represents external disturbances and unmodeled dynamics, and $f, b : \mathbb{R}^n \to \mathbb{R}$ are both nonlinear functions.

In respect of the dynamic system presented in Eq. (1), the following assumptions will be made:

Assumption 1. *The function f is unknown but bounded by a known function of $\mathbf{x}$, i.e. $|\hat{f}(\mathbf{x}) - f(\mathbf{x})| \leq F(\mathbf{x})$ where $\hat{f}$ is an estimate of f.*

Assumption 2. *The input gain b is unknown but positive and bounded, i.e. $0 < b_{\min} \leq b(\mathbf{x}) \leq b_{\max}$.*

Assumption 3. *The disturbance d is unknown but bounded, i.e. $|d| \leq \delta$.*

The proposed control problem is to ensure that, even in the presence of external disturbances and modeling imprecisions, the state vector $\mathbf{x}$ will follow a desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$ in the state space.

Regarding the development of the control law the following assumptions should also be made:

Assumption 4. *The state vector $\mathbf{x}$ is available.*

Assumption 5. *The desired trajectory $\mathbf{x}_d$ is once differentiable in time. Furthermore, every element of vector $\mathbf{x}_d$, as well as $x_d^{(n)}$, is available and with known bounds.*

Now, let $\tilde{x} = x - x_d$ be defined as the tracking error in the variable x, and

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x}, \dot{\tilde{x}}, \ldots, \tilde{x}^{(n-1)}]$$

as the tracking error vector.

Consider a sliding surface S defined in the state space by the equation $s(\tilde{\mathbf{x}}) = 0$, with the function $s : \mathbb{R}^n \to \mathbb{R}$ satisfying

$$s(\tilde{\mathbf{x}}) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x} \tag{2}$$

or conveniently rewritten as

$$s(\tilde{\mathbf{x}}) = \mathbf{\Lambda}^{\mathrm{T}} \tilde{\mathbf{x}} \tag{3}$$

where $\mathbf{\Lambda} = [c_{n-1}\lambda^{n-1}, \ldots, c_1\lambda, c_0]$ and c_i states for binomial coefficients, i.e.

$$c_i = \binom{n-1}{i} = \frac{(n-1)!}{(n-i-1)!\, i!}\,, \quad i = 0, 1, \ldots, n-1$$

which makes $c_{n-1}\lambda^{n-1} + \cdots + c_1\lambda + c_0$ a Hurwitz polynomial.

For notational convenience, the time derivative of s will be written in the following form:

$$\dot{s} = \mathbf{\Lambda}^{\mathrm{T}} \dot{\tilde{\mathbf{x}}} = \tilde{x}^{(n)} + \mathbf{\Lambda}_u^{\mathrm{T}} \tilde{\mathbf{x}} \tag{4}$$

where $\mathbf{\Lambda}_u = [0, c_{n-1}\lambda^{n-1}, \ldots, c_1\lambda]$.

Now, let the problem of controlling the uncertain nonlinear system (1) be treated in a Filippov's way [8], defining a control law composed by an equivalent control $\hat{u} = \hat{b}^{-1}(-\hat{f} - \hat{d} + x_d^{(n)} - \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}})$ and a discontinuous term $-K\,\mathrm{sgn}(s)$:

$$u = \hat{b}^{-1}\left(-\hat{f} - \hat{d} + x_d^{(n)} - \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}}\right) - K\,\mathrm{sgn}(s) \tag{5}$$

where $\hat{d}$ is an estimate of d, $\hat{b} = \sqrt{b_{\max} b_{\min}}$ is an estimate of b, K is a positive gain and $\mathrm{sgn}(\cdot)$ is defined as

$$\mathrm{sgn}(s) = \begin{cases} -1 & \text{if} \quad s < 0 \\ 0 & \text{if} \quad s = 0 \\ 1 & \text{if} \quad s > 0 \end{cases}$$

Based on Assumptions 1–3 and considering that $\beta^{-1} \leq \hat{b}/b \leq \beta$, where $\beta = \sqrt{b_{\max}/b_{\min}}$, the gain K should be chosen according to

$$K \geq \beta\hat{b}^{-1}(\eta + \delta + |\hat{d}| + F) + (\beta - 1)|\hat{u}| \tag{6}$$

where η is a strictly positive constant related to the reaching time.

It can be easily verified that (5) is sufficient to impose the sliding condition

$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \tag{7}$$

and, consequently, the finite time convergence to the sliding surface S.

In order to obtain a good approximation to the disturbance d, the estimate $\hat{d}$ will be computed directly by an adaptive fuzzy algorithm.

The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang), whose rules can be stated in a linguistic manner as follows:

$$\textit{If } s \textit{ is } S_r \textit{ then } \hat{d}_r = \hat{D}_r \;\; ; \;\; r = 1, 2, \ldots, N$$

where S_r are fuzzy sets, whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each one of the N fuzzy rules.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}(s) = \frac{\sum_{r=1}^{N} w_r \cdot \hat{d}_r}{\sum_{r=1}^{N} w_r} \tag{8}$$

or, similarly,

$$\hat{d}(s) = \hat{\mathbf{D}}^{\mathrm{T}} \mathbf{\Psi}(s) \tag{9}$$

where, $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_N]$ is the vector containing the attributed values $\hat{D}_r$ to each rule r, $\mathbf{\Psi}(s) = [\psi_1(s), \psi_2(s), \ldots, \psi_N(s)]$ is a vector with components $\psi_r(s) = w_r / \sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule.

To ensure the best possible estimate $\hat{d}(s)$ to the disturbance d, the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{D}}} = \varphi s \mathbf{\Psi}(s) \tag{10}$$

where φ is a strictly positive constant related to the adaptation rate.

It's important to emphasize that the chosen adaptation law, Eq. (10), must not only provide a good approximation to disturbance d but also assure the convergence of the state variables to the sliding surface S, for the purpose of trajectory tracking.

Theorem 1. *Consider the uncertain nonlinear system (1) and assumptions 1–5. Then, the controller defined by (5), (6), (9) and (10) ensures the convergence of the states to the sliding surface S and to the desired trajectory.*

Proof. Let a positive-definite function V be defined as

$$V(t) = \frac{1}{2}s^2 + \frac{1}{2\varphi}\mathbf{\Delta}^{\mathrm{T}}\mathbf{\Delta} \tag{11}$$

where $\mathbf{\Delta} = \hat{\mathbf{D}} - \hat{\mathbf{D}}^*$ and $\hat{\mathbf{D}}^*$ is the optimal parameter vector, associated to the optimal estimate $\hat{d}^*(s)$. Thus, the time derivative of V is

$$\begin{aligned}
\dot{V}(t) &= s\dot{s} + \varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}} \\
&= (\tilde{x}^{(n)} + \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}})s + \varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}} \\
&= (x^{(n)} - x_d^{(n)} + \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}})s + \varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}} \\
&= \left(f + bu + d - x_d^{(n)} + \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}}\right)s + \varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}} \\
&= \Big[f + d + b\hat{b}^{-1}(-\hat{f} - \hat{d} + x_d^{(n)} - \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}}) \\
&\quad - bK\,\mathrm{sgn}(s) - (x_d^{(n)} - \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}})\Big]s + \varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}}
\end{aligned}$$

Defining the minimum approximation error as $\varepsilon = \hat{d}^*(s) - d$, recalling that $\hat{u} = \hat{b}^{-1}(-\hat{f} - \hat{d} + x_d^{(n)} - \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}})$, and noting that $\dot{\mathbf{\Delta}} = \dot{\hat{\mathbf{D}}}$, $f = \hat{f} - (\hat{f} - f)$ and $d = \hat{d} - (\hat{d} - d)$, $\dot{V}$ becomes:

$$\begin{aligned}
\dot{V}(t) &= -\Big[(\hat{f} - f) + \varepsilon + (\hat{d} - \hat{d}^*) + \hat{b}\hat{u} - b\hat{u} \\
&\quad + bK\,\mathrm{sgn}(s)\Big]s + \varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}} \\
&= -\Big[(\hat{f} - f) + \varepsilon + (\hat{\mathbf{D}} - \hat{\mathbf{D}}^*)^{\mathrm{T}}\mathbf{\Psi}(s) + \hat{b}\hat{u} \\
&\quad - b\hat{u} + bK\,\mathrm{sgn}(s)\Big]s + \varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}} \\
&= -\Big[(\hat{f} - f) + \varepsilon + \hat{b}\hat{u} - b\hat{u} + bK\,\mathrm{sgn}(s)\Big]s \\
&\quad + \varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\left(\dot{\hat{\mathbf{D}}} - \varphi s\mathbf{\Psi}(s)\right)
\end{aligned}$$

Thus, by applying the adaptation law (10) to $\dot{\hat{\mathbf{D}}}$:

$$\dot{V}(t) = -\Big[(\hat{f} - f) + \varepsilon + \hat{b}\hat{u} - b\hat{u} + bK\,\mathrm{sgn}(s)\Big]s$$

Furthermore, considering assumptions 1–3, defining K according to (6) and verifying that $|\varepsilon| = |\hat{d}^* - d| \leq |\hat{d} - d| \leq |\hat{d}| + \delta$, it follows

$$\dot{V}(t) \leq -\eta|s| \tag{12}$$

which implies $V(t) \leq V(0)$ and that s and $\mathbf{\Delta}$ are bounded. Considering Assumption 5 and Eq. (4), it can be easily verified that $\dot{s}$ is also bounded.

Integrating both sides of (12) shows that

$$\lim_{t\to\infty}\int_0^t \eta|s|\,d\tau \leq \lim_{t\to\infty}\,[V(0) - V(t)] \leq V(0) < \infty$$

Therefore, it follows from Barbalat's lemma that $s \to 0$ as $t \to \infty$, which ensures the convergence of the states to the sliding surface S and to the desired trajectory. □

However, the presence of a discontinuous term in the control law leads to the well known chattering phenomenon. To overcome the undesirable chattering effects, Slotine [16] proposed the adoption of a a thin boundary layer, S_ϕ, in the neighborhood of the switching surface:

$$S_\phi = \left\{\tilde{\mathbf{x}} \in \mathbb{R}^n \;\middle|\; |s(\tilde{\mathbf{x}})| \leq \phi\right\}$$

where ϕ is a strictly positive constant that represents the boundary layer thickness.

The boundary layer is achieved by replacing the sign function by a continuous interpolation inside S_ϕ. It should be noted that this smooth approximation, which will be called here $\varphi(s,\phi)$, must behave exactly like the sign function outside the boundary layer. There are several options to smooth out the ideal relay but the most common choices are the saturation function:

$$\mathrm{sat}(s/\phi) = \begin{cases} \mathrm{sgn}(s) & \text{if } \left|s/\phi\right| \geq 1 \\ s/\phi & \text{if } \left|s/\phi\right| < 1 \end{cases} \tag{13}$$

and the hyperbolic tangent function $\tanh(s/\phi)$.

In this way, to avoid chattering, a smooth version of Eq. (5) can be adopted:

$$u = \hat{b}^{-1}\left(-\hat{f} - \hat{d} + x_d^{(n)} - \mathbf{\Lambda}_u^{\mathrm{T}}\tilde{\mathbf{x}}\right) - K\varphi(s,\phi) \tag{14}$$

Nevertheless, it should be emphasized that the substitution of the discontinuous term by a smooth approximation inside the boundary layer turns the perfect tracking into a tracking with guaranteed precision problem, which actually means that a steady-state error will always remain.

Remark 1. *It has been demonstrated by Bessa [2] that by adopting a smooth sliding mode controller, the tracking error vector will exponentially converge to a closed region* $\Phi = \{\tilde{\mathbf{x}} \in \mathbb{R}^n \mid |s(\tilde{\mathbf{x}})| \leq \phi$ *and* $|\tilde{x}^{(i)}| \leq \zeta_i \lambda^{i-n+1}\phi, i = 0, 1, \ldots, n-1\}$*, with* ζ_i *defined as*

$$\zeta_i = \begin{cases} 1 & \textit{for} \quad i = 0 \\ 1 + \sum_{j=0}^{i-1} \binom{i}{j} \zeta_j & \textit{for} \quad i = 1, 2, \ldots, n-1. \end{cases}$$

In the next section, an application of the proposed control scheme to an underwater robotic vehicle is introduced to illustrate the controller design method.

Depth control of remotely operated underwater vehicles

In the range of velocities in which remotely operated underwater vehicles typically operate, never exceeding 2 m/s, the hydrodynamic forces (F_h) can be approximated using the *Morison equation* [15]:

$$F_h = C_D \frac{1}{2} \rho A v|v| + C_M \rho \nabla \dot{v} + \rho \nabla \dot{v}_w \tag{15}$$

where v and $\dot{v}$ are, respectively, the relative velocity and the relative acceleration between rigid-body and fluid, $\dot{v}_w$ is the acceleration of underwater currents, A is a reference area, ρ is the fluid density, ∇ is the fluid's displaced volume, C_D and C_M are coefficients that must be experimentally obtained.

The last term of Eq. (15) is the so-called *Froude-Kryloff force* and will not be considered in this work due the fact, that at normal working depths, the acceleration of the underwater currents is negligible. In this way, the coefficient $C_M \rho \nabla$ of the second term will be called *hydrodynamic added mass*. The first term represents the nonlinear hydrodynamic quadratic damping.

The equations of motion for underwater vehicles can be presented with respect to an inertial reference frame or to a body-fixed reference frame, Fig. 1. For control purposes, the dynamic model of underwater vehicles are commonly expressed with respect to the inertial reference frame by the position/attitude vector $\mathbf{x} = [x, y, z, \alpha, \beta, \gamma]$.

In the particular case of remotely operated vehicles, the distance between buoyancy and gravity centers is usually large enough to keep the roll (α) and pitch (β) angles small, i.e. $\alpha \approx 0$ and $\beta \approx 0$. Besides the self-stabilizing property, this design characteristic allows the vertical motion (heave) of the vehicle to be considered decoupled from the motion in the horizontal plane. This simplification can be found in the majority of works presented in the specialized literature [7, 10, 11, 12, 13, 18, 20]. So, with this in mind and considering Morison equation, the vertical motion along z-axis can be described by

$$m\ddot{z} + c\dot{z}|\dot{z}| + d = u \tag{16}$$

where u is the control input (thrust force), d the disturbance caused by external forces, $c = \frac{1}{2} C_D \rho A$ the coefficient of the hydrodynamic quadratic damping and m represents vehicle's mass plus the hydrodynamic added mass.

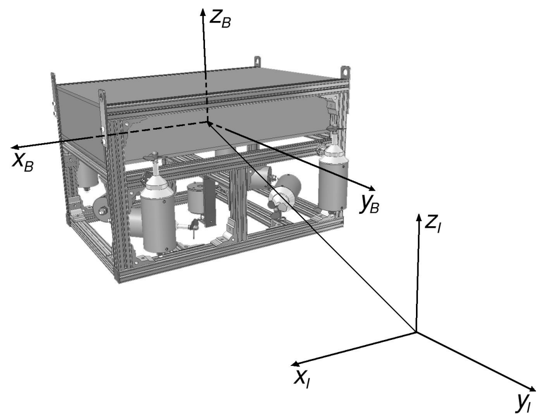

Figure 1: Underwater vehicle with both inertial and body-fixed reference frames.

In this way, based on Eqs. (6), (9), (10), (13) and (14) and considering that $s(\tilde{z}, \dot{\tilde{z}}) = \dot{\tilde{z}} + \lambda \tilde{z}$, $|\hat{c} - c| \leq \varsigma$ and $\mu^{-1} \leq \hat{m}/m \leq \mu$, where $\mu = \sqrt{m_{\max}/m_{\min}}$, the following control law can be proposed to deal with the dynamic model presented in Eq. (16).

$$u = \hat{c}\dot{z}|\dot{z}| + \hat{d} + \hat{m}(\ddot{z}_d - \lambda \dot{\tilde{z}}) - K \operatorname{sat}(s/\phi) \tag{17}$$

where the control gain is defined according to

$$K \geq \hat{m}\mu\eta + \varsigma \dot{z}^2 + \delta + |\hat{d}| + \hat{m}(\mu - 1)|\ddot{z}_d - \lambda \dot{\tilde{z}}| \tag{18}$$

For a more detailed discussion about the development of adaptive sliding mode controllers for remotely operated underwater vehicles see [3, 4].

Simulation results

The simulation studies were performed with an implementation in C, with sampling rates of 500 Hz for control system and 1 kHz for dynamic model. The differential equations of the dynamic model were numerically solved with a fourth order Runge-Kutta method. Concerning the fuzzy system, triangular and trapezoidal membership functions were adopted for S_r, with the central values defined as $C = \{-5.0 ; -1.0 ; -0.5 ; 0.0 ; 0.5 ; 1.0 ; 5.0\} \times 10^{-3}$. It is also important to emphasize, that the vector of adjustable parameters was initialized with zero values, $\hat{\mathbf{D}} = \mathbf{0}$, and updated at each iteration step according to the adaptation law, Eq. (10).

In order to evaluate the control system performance, two different numerical simulations were performed. The obtained results are presented in Figs. 2–6.

In the first case, it was considered that the model parameters, m and c, were perfectly known. Regarding controller and model parameters, the following values were chosen $\hat{m} = m = 50$ kg, $\hat{c} = c = 250$, $\mu = 1$ and $\varsigma = 0$. The disturbance force was chosen to vary in the range of ± 5 N (see Fig. 5). The other used parameters were $\delta = 5$, $\lambda = 0.6$, $\eta = 0.1$, $\phi = 0.01$ and $\gamma = 150$. Figures 2–5 gives the corresponding results for the tracking of $z_d = 0.5[1 - \cos(0.1\pi t)]$, considering that the initial state coincides with the initial desired state, $\tilde{\mathbf{z}}(0) = [\tilde{z}(0), \dot{\tilde{z}}(0)] = \mathbf{0}$.

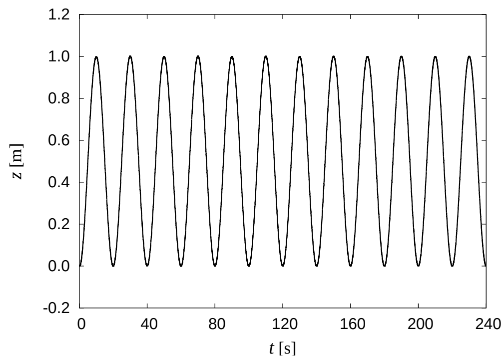

Figure 2: Depth tracking with known parameters.

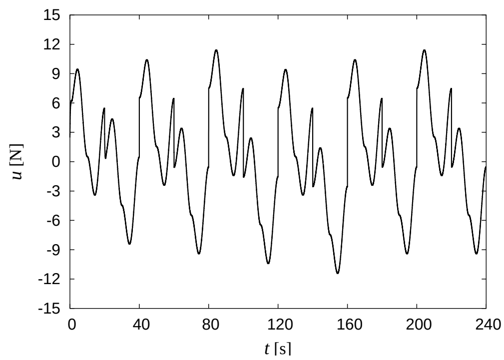

Figure 3: Control action with known parameters.

As observed in Figs. 2–5, even in the presence of external disturbances, the adaptive fuzzy sliding mode controller (AFSMC) is capable to provide the trajectory tracking with a small associated error and no chattering at all. It can be also verified that the proposed control law provides a smaller tracking error when compared with the conventional sliding mode controller (SMC), Fig. 4. The improved performance of AFSMC over SMC is due to its ability to recognize and compensate for external disturbances, Fig. 5. The AFSMC can be easily converted to the classical SMC by setting the adaptation rate to zero, $\varphi = 0$.

In the second simulation study, the parameters for the controller were chosen based on the assumption that exact values are not known but with a maximal uncertainty of $\pm 10\%$ over previous adopted values,

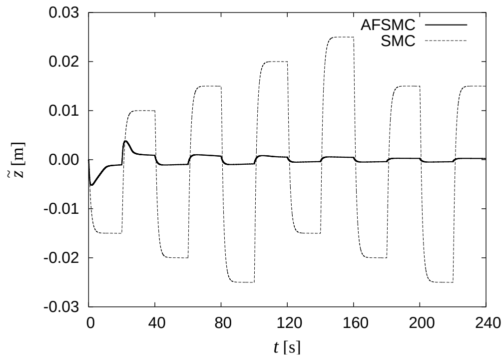

Figure 4: Tracking error with known parameters.

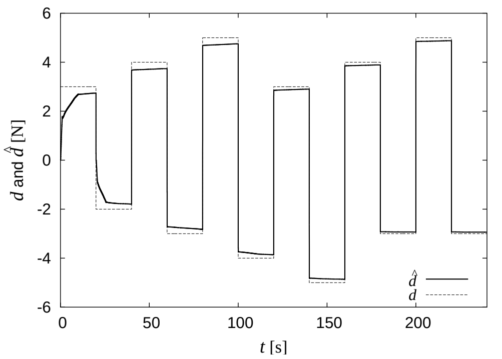

Figure 5: Convergence of $\hat{d}$ with known parameters.

$\hat{m} = 49.75$ kg, $\hat{c} = 250$, $\mu = 1.1$ and $\varsigma = 25$. For the dynamic model, it was selected $m = 55$ kg and $c = 275$. The other parameters, as well as the disturbance force and the desired trajectory, were defined as before. Despite the external disturbance forces and uncertainties with respect to model parameters, the AFSMC allows the underwater robotic vehicle to track the desired trajectory with a small tracking error (see Fig. 6). As before, the improved performance of the AFSMC over the uncompesated counterpart can be clearly ascertained.

Concluding remarks

In this paper, an adaptive fuzzy sliding mode controller was developed to deal with uncertain single-input–single-output nonlinear systems. To enhance the tracking performance inside the boundary layer, the adopted strategy embedded an adaptive fuzzy algorithm within the sliding mode controller for uncertainty/disturbance compensation. Using Lyapunov's direct method and Barbalat's lemma, the convergence properties were analytically proven. To evaluate the control system performance, the proposed scheme was applied to the depth control of a remotely operated underwater vehicle. Through numerical simulations,

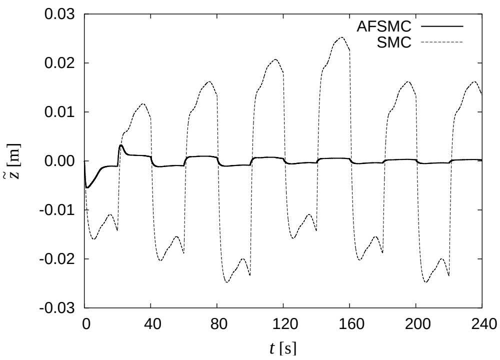

Figure 6: Tracking error with uncertain parameters.

the improved performance over the conventional sliding mode controller was demonstrated.

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